AMPS Technologies Company, http://www.ampstech.com
1310 Old Freeport Road, Suite 38121, Pittsburgh, PA 15238, USA
Phone: 412-963-1753, email: info@ampstech.com

Sefea™: Strain-Enriched Finite Element Analysis – A new generation of FEA, theory and benchmarks

Executive Summary

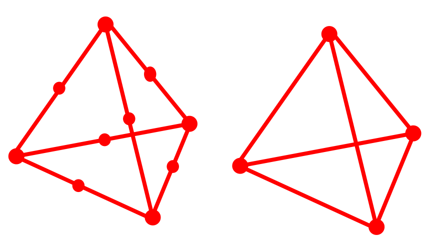
- Sefea is one of the newest generations of enriched finite element methods.
- Developed specifically for low-order 4-node tetrahedron and 3-node triangle in the CAE environment, Sefea achieves the accuracy of 2nd-order elements such as 10-node tetrahedron at the low-order element computing cost and robustness without the 2nd-order element side-node noise.
- Sefea can withstand user abuse with tolerance for mesh density variation and distortion, which commonly occur in an automatic tetrahedron mesh generator in CAE.
- Unified enrichment method suitable for fully coupled stress, thermal, fluid, and EM physics in an integrated multiphysics formulation.

Sefea Basic Theory

Finite element method (FEM) has been one of the backbones in stress and thermal analyses since the late 1960’s and has expanded further into fluid flow, electromagnetic, and many other areas. Today, FEM software is now routinely used in corporate R&D and in general CAE applications on a daily basis.

Many methods, such as Boundary Element Method (BEM) in the 80’s and the Element-Free Galerkin Method (EFGM) in the 90’s, have proposed new algorithms to solve specific focuses but were not as popular for general applications. However, these valuable researches have inspired further FEM development into areas such as Extended FEM and Enriched FEM, with the goal to solve specific problems that are difficult to tackle in the traditional FEM.

Sefea came from the Enriched FEM family and focuses on solving problems in the increasing realistic CAE simulations performed daily by CAD users with minimum to medium FEM know-how, as well as by experienced analysts drawn into the convenient solid modeling technology. However, in such CAE environments, only the tetrahedral element generation is robust and reliable enough for general geometry. The low-order 4-noded tetrahedron (TET4) is robust enough to solve general problems but is considered too stiff; only the 2nd-order 10-node tetrahedron (TET10) is accurate enough for general analysis, although it still exhibits deficiency for general nonlinear dynamic problems.


TET10 (left) and TET4 (right) elements

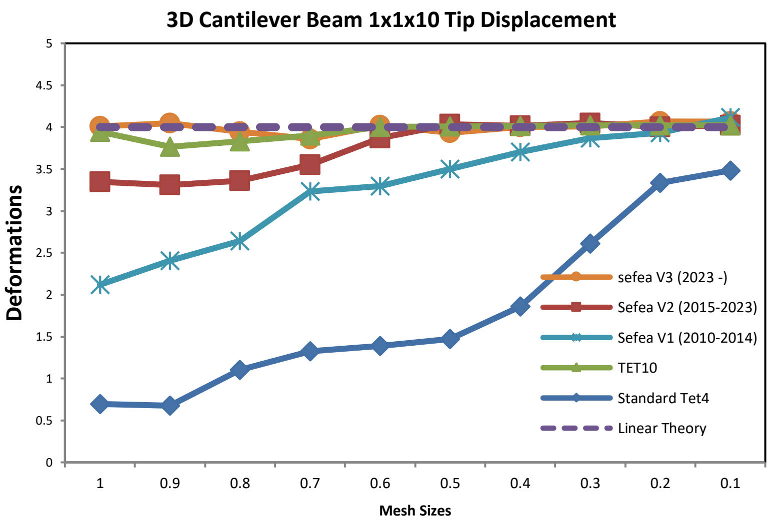
The poor performance of the TET4 element is due to the inability to represent all types of deformation strain, or the gradient of displacement. The constant strain behavior in the TET4 cannot represent the physics well, as compared with the more flexible tri-linear strain in TET10.

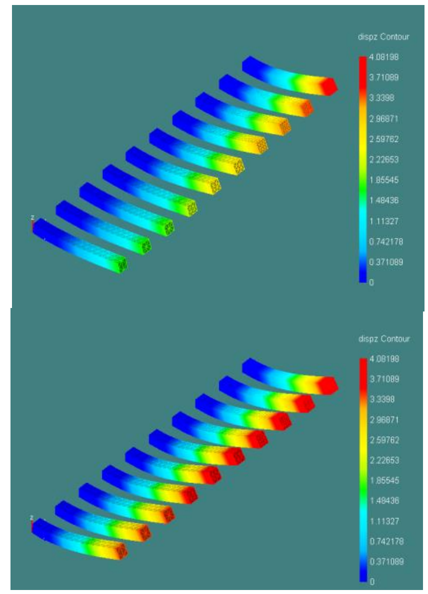
The deformation strain is usually described in two parts: the dilatational strain that describes the volumetric change, and the deviatoric strain from the shear deformation. Inspired by the strain projection method proposed by Hughes, and recognizing that the constant dilatational strain in TET4 is the culprit of locking, we incorporate the more accurate dilatational strain formulation from each TET4 corner node as proposed in EFGM, while keeping the deviatoric strain from the FEM. Effectively, Sefea “enriches” the TET4 constant strain into an equivalent tri-linear strain while keeping the TET4 reliable shearing behavior without using additional nodes, and thus there are no additional equations to solve.

Using such enrichment methods, Sefea delivers a new low-order element family in the same robust FEM tool that has been used for more than a half century, and it provides accuracy improvement from the mesh-free method.

3D Sefea Linear Elastic Benchmark

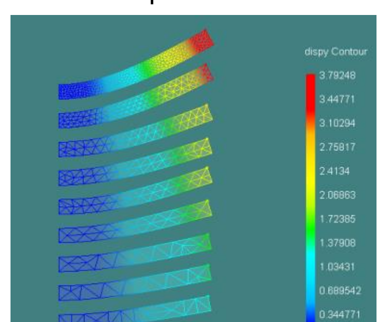
To demonstrate the Sefea formulation convergence behavior, 10 cantilever beams of size 1x1x10, Young's modulus=1000, Poisson's ratio=0.35 with a free-end distributed shear loading of one are used for this test. Each beam is meshed using an average TET4 mesh size ranging from 1 to 0.1.





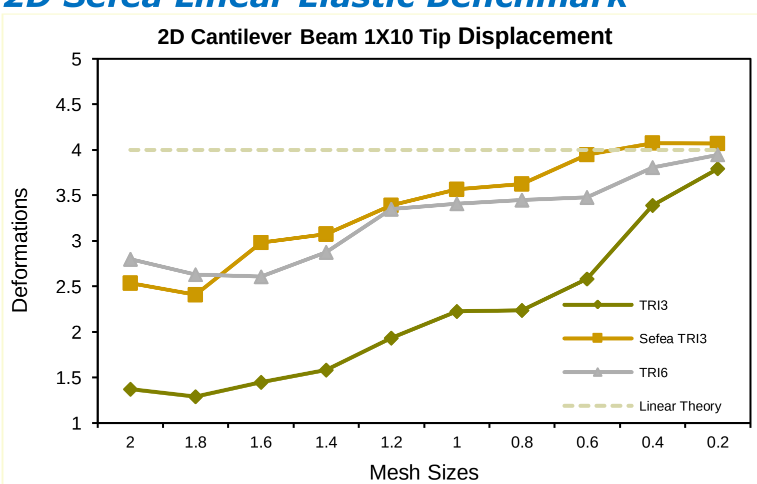
TET4 (top) and Sefea TET4 (bottom) and elements deformation contour

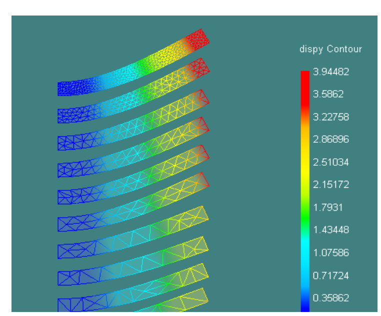
As shown in the picture and the chart, Sefea TET4 element model converges very quickly to the theoretical solution once the mesh size is refined to about half of the beam depth, and usually this mesh size criterion is a common practice in CAE applications. TET10 element modeling, even with coarse mesh, behaves well in this static benchmark since the quadratic displacement describes this bending behavior well. The standard TET4 element exhibits a locking behavior as expected and managed to reach engineering accuracy at mesh size of 0.1.



Standard TRI3 elements

2D Sefea Linear Elastic Benchmark





Sefea TRI3 elements

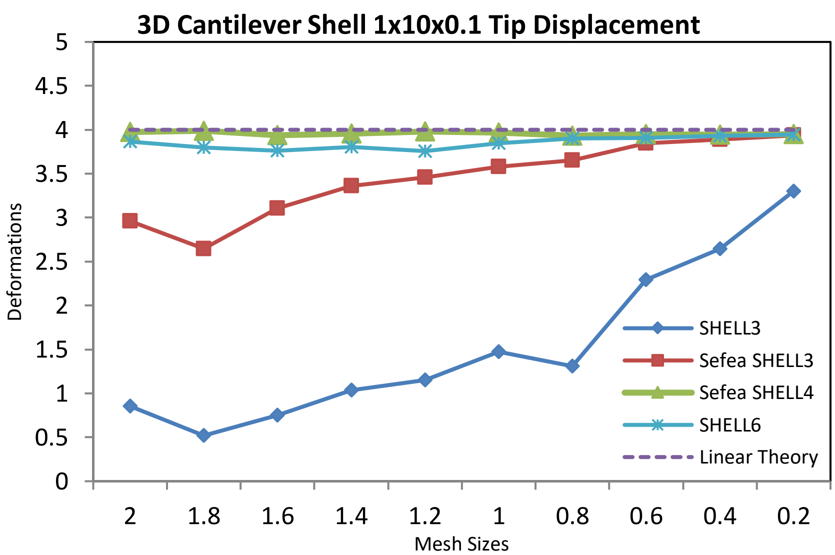
Similar to the 3D benchmark, ten 2D plane-stress cantilever beams of size 1x10, Young's modulus=1000, Poisson's ratio=0.35 with a free-end distributed shear load=1 are used to demonstrate the Sefea 2D behavior. Each beam is meshed with an average triangle mesh size from 2 to 0.2, with increment of 0.2. The mesh variations and sizes are even coarser than the 3D case and are usually not recommended, except for the demonstration of theoretical benchmarks.

Again, the results demonstrate Sefea low order TRI3 element model converges very quickly and stays slightly closer to the theoretical solution than the 2nd-order TRI6 element model, except in the extremely coarse mesh condition (mesh > 1.6). In our observations, Sefea in 2D domain converges slightly faster than 3D domain and is resistive to mesh distortion, as can be observed from the flatter curve. The 2nd-order TRI6 formulation deteriorates from the 3D behavior due to 2D plane stress constrain and has even less advantage as compared with Sefea method.

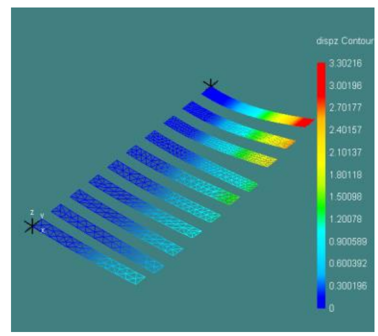
The standard TR3 element model failed to reach the theoretical solution even at the finest mesh density in the range studied. The much wider usable mesh range and resistance to mesh distortion is a critical factor in selecting FE tools in CAE applications.

3D Sefea Shell Linear Elastic Benchmark

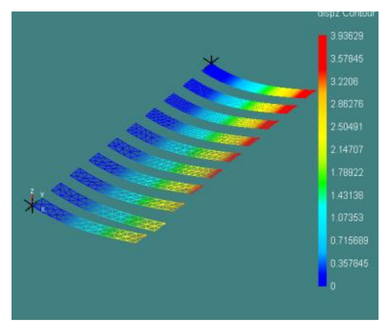
For 3D shell behavior, the same ten cantilever beams of size 1x10 with thickness of 0.1, Young's modulus=1000, Poisson's ratio=0.35 with a distributed shear loading of 0.001 at the free-end are used for this benchmark. Each beam is meshed with an average triangle mesh size from 2 to 0.2, with increment of 0.2.



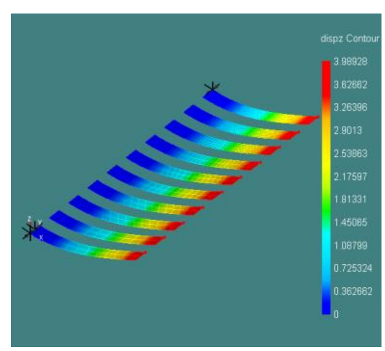
Again, Sefea SHELL3 performed much better than the low order SHELL3. The Sefea SHELL4 element performs even better than the 2nd-order SHELL6



Shell3 elements



Sefea SHELL4 elements



Sefea SHELL3 elements

Sefea Element Patch Test

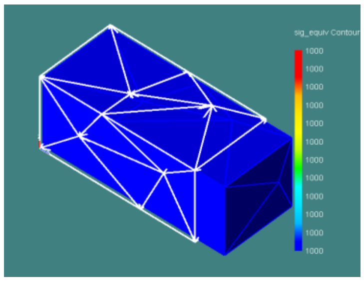
Patch test is one of the standard tests to ensure the finite element solution convergence, ensuring that the exact solution can be duplicated when infinite numbers of elements are used.

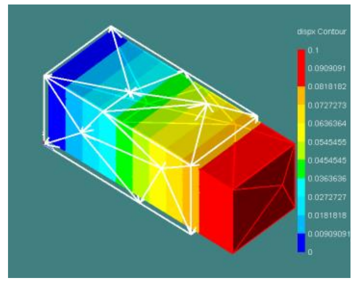
Two types of patch tests have been performed. The first patch test applies a fixed deformation to check the uniform strain field, and the second test uses an external uniform pressure loading to test the stress variation. A minimum of one completely constraint free element is included in the domain, and both 2D and 3D domains are tested using Sefea low order element formulation. In both cases, both 2D triangle Sefea TRI3 and 3D Sefea TET4 show uniform strain and stress perfectly.

Uniform Stress

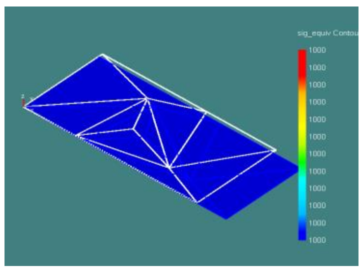
Uniform Deformation/Strain

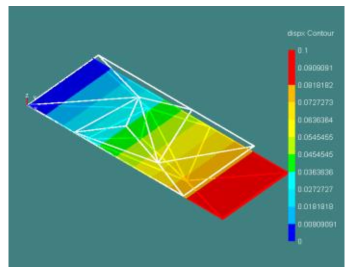
3D Sefea TET4 elements





2D Sefea TRI3 elements



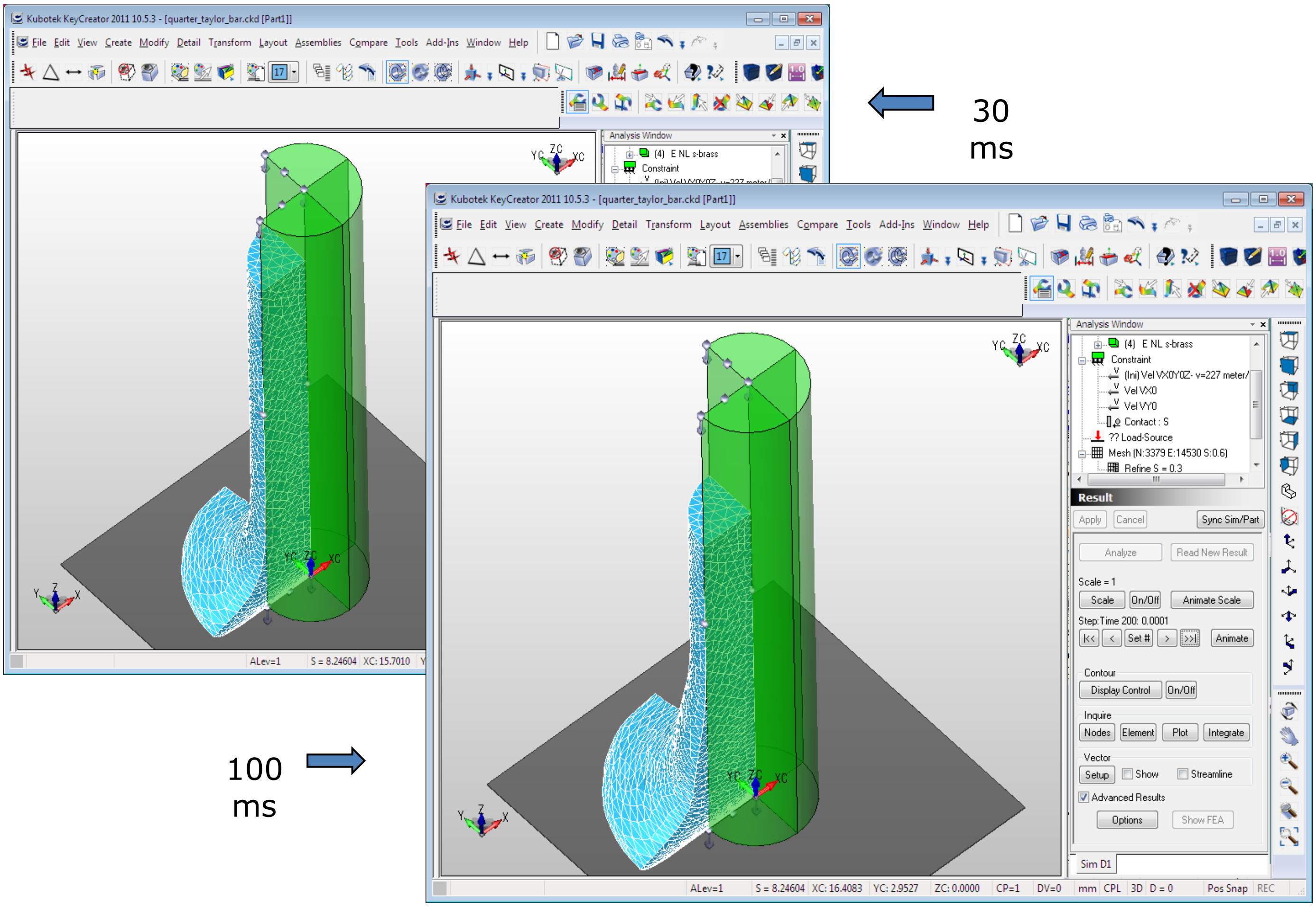


Taylor Bar Impact Benchmark for Elasto-Plastic Large Deformation Physics

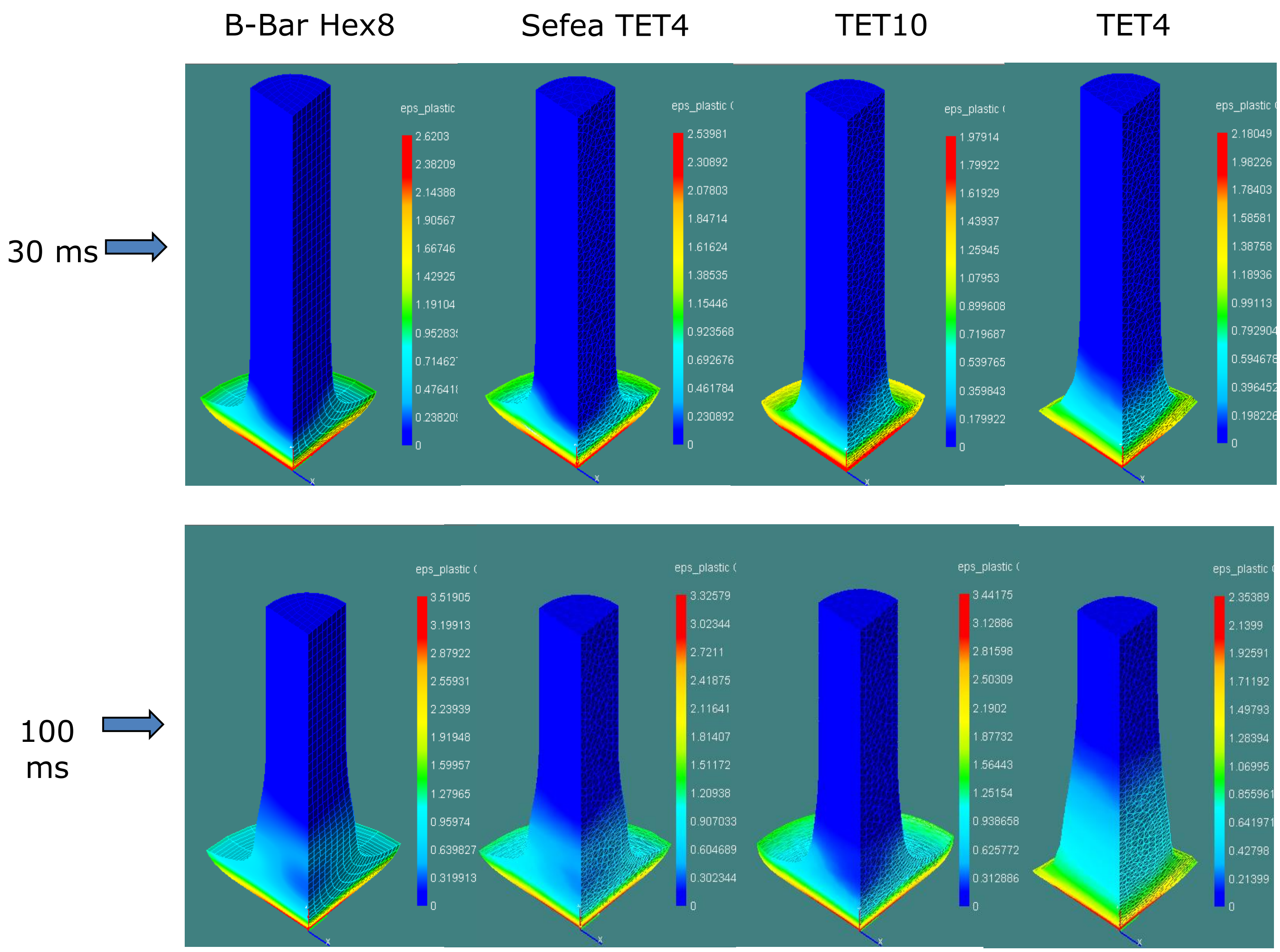
The Taylor bar impact test is popular for nonlinear large deformation plasticity benchmark. The bar has an initial length of 32.4mm with an initial radius of 3.2 mm, Young's modulus=1.17e11 Pa, Poisson's ratio=0.35, initial yield strength=4.0e8 Pa, hardening modulus=1.0e8 Pa, and mass density of 8.93e3 Kg/m^3. The bar has an initial impact speed of 227m/sec to a rigid frictionless wall. Initial time step of 5.0e-7 seconds is taken for 200 steps. Quarter symmetry model is adopted with an average mesh size of 0.6mm. No ALE mesh adjustment is used for benchmark and accuracy purposes.

The impact results at 30ms are available in many publications. Sefea TET4 and B-bar HEX8 results compared well in deformation and plastic strain distribution. The standard TET4 element is overly rigid, even at the refined 0.2 mm mesh size and the costly 2.0e-7 time step size, as expected. The Sefea TET4 behaves almost the same as the B-Bar hexahedron elements. The TET10 result is similar to those of the HEX8 and Sefea, but with a lower plastic strain at 30ms and a slightly different plastic strain distribution at 100ms.

In Sefea TET4, corner nodes Lobotto quadrature method is used. When the large nonlinear effects are near the surface as in this case, it is more accurate than the Gaussian method that has sampling points in the inner areas of the TET4, HEX8 or TET10 element. Note that in TET10 model, the number of nodes is almost 7 times larger than in the Sefea and TET4, even though both have the same number of elements. In general, the 2nd order family elements usually have 5 to 10 times more equations than the equivalent first order family elements and require much longer solution time. In this small problem, the equation solver time is less than 30% of the total solution time, and the runtime difference is only about 1217 vs. 550 seconds for iterative solver (or 2158 vs. 693 for direct solver). But for general applications, the solution time for TET10 will ramp up quickly and could be cost prohibitive for large problems or nonlinear problems with many refined load increments.



Types	Total Elements	Total Nodes	Iterative /direct solver Time (sec)	Peak plastic strain (30 ms)	Peak plastic strain (100ms)	Rod deformation (mm)	Final head radius (mm)
Hex8	2400	3112	476 / 523	2.620 /2.934	3.519 /3.395	10.24	7.83
Sefea TET4	14544	3359	550 / 693	2.540 /2.540	3.325 /2.756	10.20/10.4	7.89 /7.28
Tet10	14544	22778	1217 / 2158	1.979	3.444	9.72	7.99
Tet4	14544	3359	479 / 583	2.180	2.354	10.50	5.69

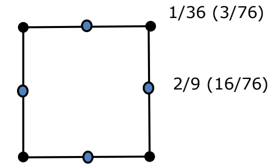


Sefea vs. 2nd-Order Element

The 2nd-order element, of either Lagrangian or Serendipity family, usually is considered more accurate than the first order element. However, many analysts or experienced users have chosen other lower order elements such as HEXA8 brick element for more stable solution, especially in nonlinear dynamic applications. These HEXA8 brick elements have several different enhanced formulations such as one-point-integral with hourglass mode control, or B-Bar formulation. However, automatic hexahedron meshing currently still generates distorted mesh in many general applications and is generally not usable except in some semi-automatic tools for experienced users. In most CAE applications, automatic tetrahedron meshing has been the main tool, and the 2nd-order TET10 element is the workhorse for most analyses.

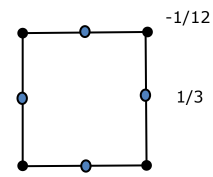
When the solution of a problem is smooth, as shown in the simple cantilever beam, usually a small number of 2nd-order elements such as TRI6 or TET10 can be used to solve the problem quickly. However, in many situations, refined mesh is needed around the model boundary with load/fixity conditions or small geometry features in order to capture sharp solution variations. The refinement is essentially needed when dealing with nonlinear material such as plasticity or in heterogeneous materials with large deformation.

However, the mesh refinement quickly increases the total node count in the 2nd-order case and usually becomes an issue for solvability. For a problem with the same number of TET4 and TET10 elements, the total node count in the TET10 model is usually 5 to 10 times greater than that in the lower order Sefea TET4, and the solution time goes up very quickly, even with the fastest iterative solver. Sefea elements are usually the best choice for most problems because they are equipped with 2nd order accuracy, yet only the cost of first order elements.

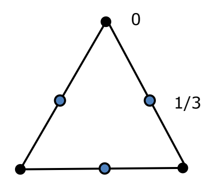


QUAD8 HRZ lumped mass ratio based on 2x2 integral vs. 3x3 integral (in parenthesis)

Furthermore, the mid-side nodes in the 2nd-order family are generally the source of solution oscillation, especially in dynamic stress or in transient thermal analysis. When the lumped mass matrix is needed for faster dynamic analysis, the lump mass matrix could be dependent on the quadrature scheme or lumping algorithm, e.g., HRZ lumping, and it could have slightly different results. In some situations, such as optimal lumping method for the high order elements, it will produce zero or even negative lumped mass at the corner nodes. Even when the consistent mass matrix is used, the matrix is also pending on the integration scheme and order. As a result, these side nodes usually create undesirable noises in the dynamic analysis. This is usually not realized in the static analysis because the mass matrix is not used, unless specific mass related body force exists.



QUAD8 optimal mass lumping ratio

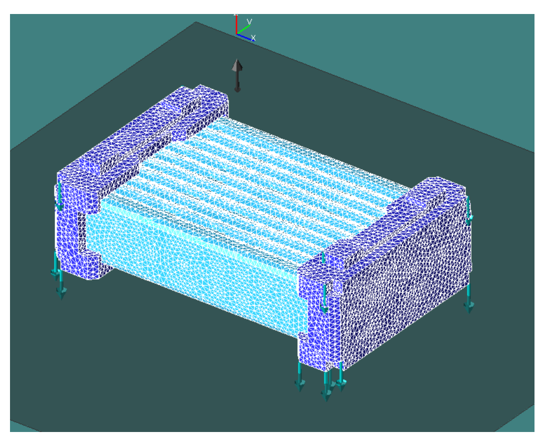


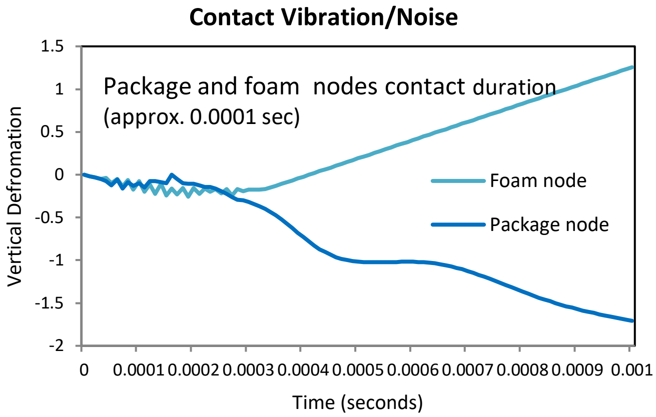
TRI6 HRZ lumped mass ratio based on 4-point integral

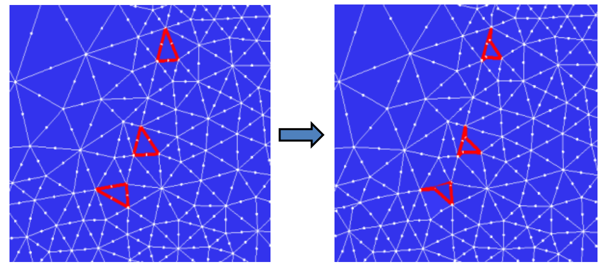
A simple modal vibration analysis of 2nd-order elements shows that these side nodes usually produce additional lower vibration modes since these side nodes have lower frequency spectrum as compared to the corner nodes, and generally these modes are more mathematical than physical. When used in general large deformation dynamic/impact/contact analyses, the variations of corner node to side node dynamic stiffness pattern usually trigger more iterations and are harder to converge, and they may even lead to undesirable solutions. This is one of the main reasons why the 2nd-order elements are rarely used in general nonlinear contact/impact/crash analysis.

Sefea vs. 2nd-Order Element in dynamic Impact/Contact analysis

A packaging drop test simulation is a good example to demonstrate the 2nd-order element mid-side node noise effect and the Sefea element's stability. The package, about 6x16x24 cm, is wrapped in a foam material at both ends, and dropped from approximately 0.6 meters to a rigid frictionless ground. The package and foam have pressure-wave speeds of approximately 2700 m/sec and 1200 m/sec, respectively. Without considering material damping, the impact will create a shock wave and white noise vibration at the ground contact points and between the package and the foam.



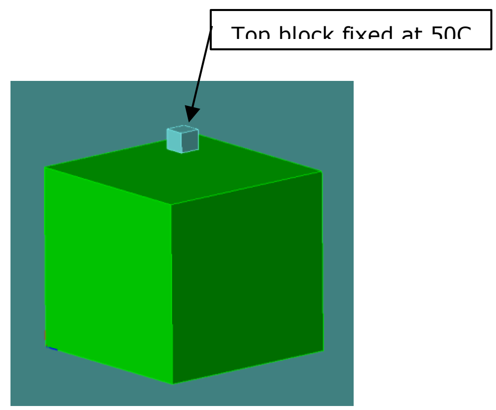




TET10 element faces on contact surface before impact(left) and the side-node noise distortion failure at first impact

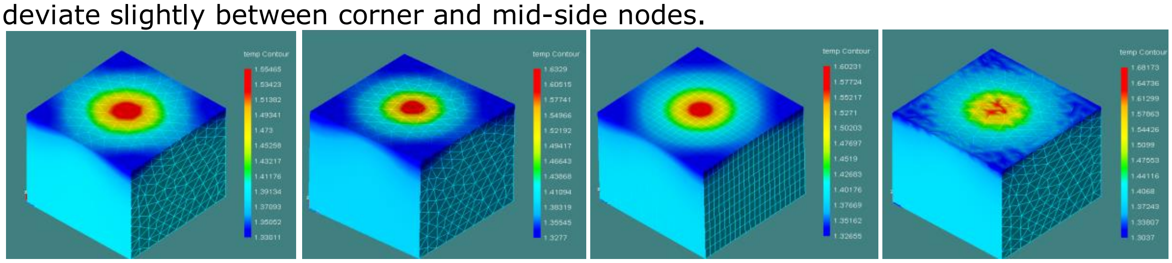
Using a 10ms time step, the 2nd-order element failed during first impact for elements between the package and foam contact interface. The Sefea TET4 survived the test, and the deformation histogram shows the heavy contact vibration between the package and the foam. The picture of the failed 2nd-order element distortion shows the side-node excited by the impact, which created heavy distortion and crashed the analysis. Although these vibrations are usually more mathematical than physical because most materials exhibit actual damping, a robust dynamic method using Sefea element can usually tolerate much higher impact speed and produce more realistic behavior.

Sefea Thermal Analysis Benchmark



To demonstrate the accuracy of Sefea method in thermal applications, a radiation heat transfer between two blocks is modeled using Sefea TET4, TET4, and the 2nd-order TET10 formulation. A small 1x1x1 block is fixed at temperature 50C and held at distance of one above a lower block of 9x9x9 with the initial temperature of zero. The steady-state temperature of the bottom block is computed based on the radiation heat transfer from the top block using view factor matrix. Emissivity is 0.8 for all surfaces.

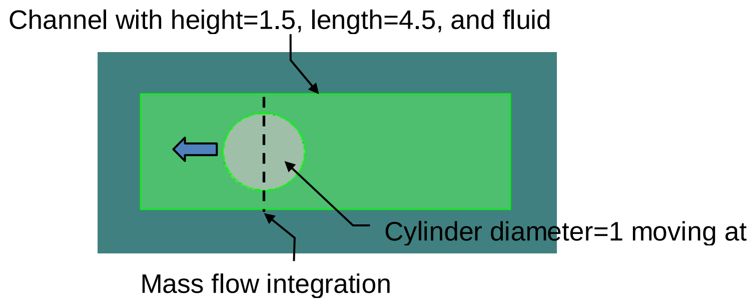
The temperature result of the larger block is shown below. The standard TET4 underestimated the temperature rise and did not show the concentric temperature ring well. Even with the same TET4 mesh, Sefea TET4 compared well with almost twice as refined brick HEX8 element results. The last 2nd-order TET10 model, with the same number of elements, shows slight variation between the TET10 corner nodes and side nodes, although the overall pattern is the same. Such wavy pattern across the TET10 element surface is due to the radiation flux integration incompatibility with the 2nd order behavior of the element, which can cause the temperature solution to deviate slightly between corner and mid-side nodes.



Thermal radiation temperature result TET4, Sefea TET4, HEX8, and TET10 (from left to right)

Sefea CFD Benchmark

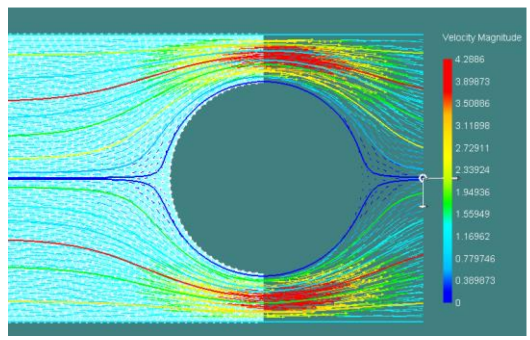
The moving cylinder in a confined channel flow problem is a popular benchmark for mass conservation in finite element Computational Fluid Dynamics (CFD). The benchmark focuses on comparing the total mass flow passing through each cross section.



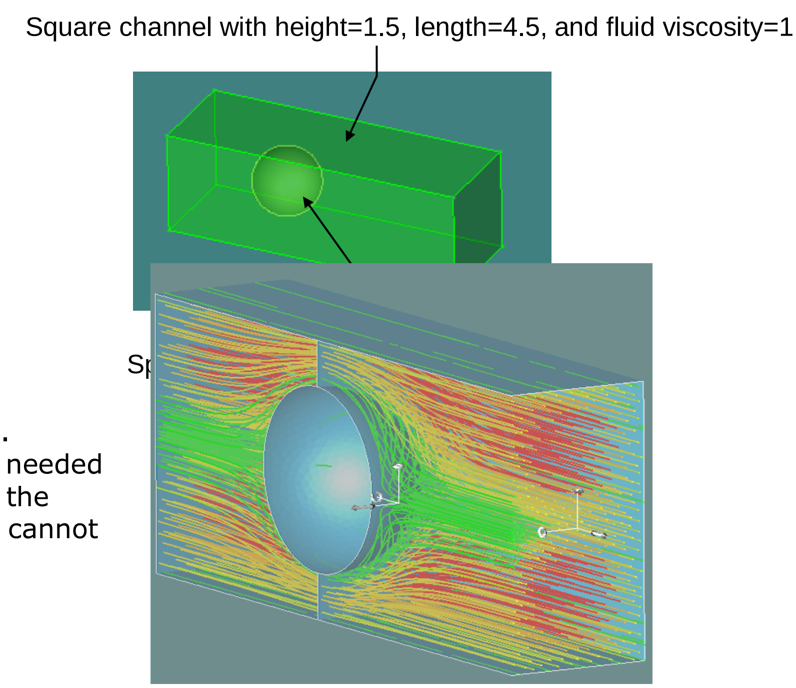
The mass conservation error at the narrowest flow section is computed by integrating the total flow passing the cross section plane. Automatic meshing with an average mesh size of 0.04 is used for all types of meshes.

The standard low order QUAD4 and TRI3 elements have much lower velocity profile, with high mass conservation errors. The Sefea QUAD4 and TRI3 elements produced almost the same result as the 2nd-order QUAD9 and TRI6 elements. Sefea TRI3 model, at the same element density but with only about ¼ of the node size, performs even slightly better in capturing the peak velocity and along the boundary.

Element Type	Total Elements	Total Nodes	Mass Conservation error (percentage)
QUAD4	3978	4180	21.57
QUAD9	3978	16316	0.47
Sefea QUAD4	3978	4180	0.02
TRI3	10028	5216	36.31
TRI6	10028	20460	0.38
Sefea TRI3	10028	5216	0.32



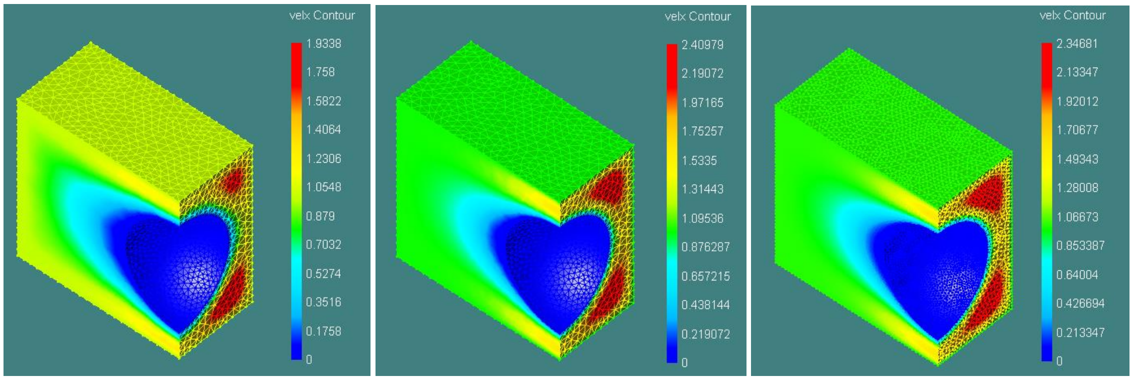
The 3D version of the CFD benchmark compares the mass conservation of a moving ball in the channel flow problem. Automatic meshing with an average mesh size of 0.07 is used for all mesh types. Half symmetry is used.



The mass flow error at the narrowest flow section is compared. The standard TET4 has an unacceptably high mass conservation error and should not be used. The Sefea TET4 elements produced slightly better mass conservation result than TET6, as in the 2D case. In general applications, fewer TET10 elements can achieve the same results produced by low order elements. However, in this case, proper mesh refinement is needed around the flow boundary laminar zone. Without the refined meshes, as in many situations, the model cannot capture the physics well.

Element Type	Total Elements	Total Nodes	Total Equations	Run Time (sec) Iter/Direct solver	Mass Conservation Error (percentage)
TET4	132883	27116	189812	17 / 85	34.87
TET10	132883	194293	1360051	328 / 5412*	2.36
Sefea TET4	132883	27116	189812	59 / 475	1.11

*Sparse direct solver uses memory paging in 32-bit 4G space.



Velocity profile for TET4, Sefea TET4 and TET10 elements (from left to right) along the channel direction

The equation solver time in this case is about 80% of the total run time. This is typical in most general CAE cases. Sefea Least-Squares method uses equal-order velocity-pressure-vorticity formulation, and the generated Sefea equation matrix is much denser. Even with this denser matrix pattern, it is still running 5 times faster (59 vs. 328 seconds) than the TET10 method and with better accuracy. Direct solver is hardly used for large commercial applications, as it usually exhausts the system memory quickly as in this case, and the solution time is more than 10 times greater (5412 vs. 475 seconds).

Conclusion

Sefea formulation preserves the robustness of the low order elements such as TET4 and HEXA8 while achieving the accuracy of 2nd-order elements, without the computing costs and the mid-side-node noise and mesh distortion problems in dynamic applications.

In extremely coarse mesh condition, the current Sefea formulation cannot represent the physics well, but the solution converges very quickly with slight mesh density improvement. Nonetheless, the mesh density can be controlled automatically for general CAE users who may have less FEM knowledge. Ongoing research has indicated several potential methods to cure such deficiencies.

Due to the larger sampling space for the strain and gradient calculation, Sefea is more stable in contact, impact, and nonlinear strain that exhibits sharp solution jumps, and it can tolerate mesh density variation and distortion commonly encountered even in an advanced automatic mesh generator. The consistent enrichment algorithm for stress, thermal, fluid, and EM physics enables integrated multiphysics formulation within a single element for more realistic and advanced simulation.

For most 3D problems meshed using the same number of tetrahedrons, the total number of nodes involved is usually 5 to 10 times greater than for 2nd-order element approaches. As solution costs grow exponentially, the low computing cost allows Sefea to solve more realistic problems that were not feasible in the past.

In summary, three of the most valuable characteristics of Sefea in CAE applications are:

1. Achieving 2nd-order element accuracy while using easily generated first order tetrahedral elements.
2. The ability to use finer mesh without exponentially increasing computing cost, as is often needed in many nonlinear heterogeneous problems, and
3. Robustness without the risks of higher-order element side-node mesh distortion and noise.

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